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ESTADÍSTIKA

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ESTADÍSTIKA

ERREALITATEAREN IKERKETA OSAGAI
NAGUSIEN ANALISIAREN BIDEZ

Egile: Anjeles Iztueta



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O. SARRERA

Gizonaren adimenak, ezagutzera iristen den edozein gertaera edo bibentzia, beti, aztertzeke eta argitzeko joera du.

Esan genezake erru haundirik gabe, geure pentzakera dimentsio bakarrekoa dela, errealitatea aztertzerakoan, ikuspuntu baten, egitasuna ala faltsutasuna sendotzea, besterik ez dugula helburu tzat hartzen.

Bein bitartean, errealitatea infinitu dimentsio - dituen espazio batetako da, eta, bera aztertzerakoan, milaka ikuspun tuen, egitasuna edo faltsotasuna sendotu behar genituzke.

Guttienez, pentzakera dialektiko batez, errealita- tea aztertzean, beste ikuspuntu bat, ere kontuan hartzen dugu; kontra ko ikuspuntua.

Baina behar genukeena, "pentzakera multidialektiko" bat izango zen.

"MULTIDIMENSIONAL ANALISI METODOETAZ, ETA, GEHIEN- BAT, OSAGAI NAGUSIEN ANALISIAZ, LOR GENEZAKE, ERA ARRAZIONAL BATETAZ, GEURE PENTSAMOLDEARI, AZTERGAITZA ZAION, R^m ESPAZIO MULTIDIMENSIONAL BAT, BESTE DIMENTSI O BAKARREKO, EDO, DIMENTSI O GUTXI KO, ESPAZIO BATE- TARA, ERREDUZITZEA".

Giza zientzietan, lehen urratsa, ikergai dugun - errealitatea definitzea da.

Eguno pentzakera zientifikoa, errealitatea: "es- truktura" bat bezala definitzen da, non errealitatea osatzen duten - elementuen arteko errelazio bereziak, estrukturaren ezaugarri garran- tzituzenak bezala hartuko dira.

Estrukturak bere beitan hertsia ez izan arren, - (zeren eta lotura asko baitituzte inguruko kanpo estrukturekin), kome ni da, ikerketa bakoitzean, aztergai dugun ektura ahal dugun neu- rrian isolatzea eta definitzea.

" GERTAKIZUNEN ITSASOA ZEHARKATZEN DUTEN;
LEGE KORRONTEAK EZAGUTZEKO
MILAKA DATU ERABIL BEHARREAN AURKITZEN GARA
ETA NAHIZ DATU HOIEK BAKARKI INFORMAZIO GUTXI ERAMAN
ESTRUKTURA BAT ERATZEN DUTE
MULTIDIMENSIONAL ANALISI METODOETAZ ETA GEHIEN BAT
OSAGAI NAGUSIEN ANALISIAZ
ESTRUKTURA AZTERTU ETA DEFINI DEZAKEGU ".



1. AZTERKETAREN TAULA

I. Suposa dezagun: egin ditugula p-test x_1 x_2 -- x_p edo p-aldagaien azterketa N indibiduei buruz.

A/ Aldagaiak aurkezpenean hartzen dituzten balioak zenbatu garriak direla suposatuko dugu (unitate berdinetan ala desberdinetan - neurtuak).

Adibidez: garaiera, adina, pisua, alokairua . . . aldagai zenbatugarriak dira, unitate desberdinetan neurtuak, (cm, urte, kgr, pza . . .)

Inteligentzia, trebetasuna, mina . . . aldagai zenbatugaitzak dira.

B/ Guren aurkezpeneko N-indibiduoak equiprobableak suposatuko ditugu.

Indibiduoak izan litezke, pertsonak, herrialdeak, zabalera, geografikoak . . .

● Azterketaren erresultatuak "taula" batetan azalduko ditugu:

	1	2	J	N
x_1	x_1^1	x_2^2	x_1^j	x_1^N
x_2	x_2^1	x_2^2	x_2^j	x_2^N
x_i	x_i^1	x_i^2	x_i^j	x_i^N
x_p	x_p^1	x_p^2	x_p^j	x_p^N

"TAULA"

Non: x_i^j , " x_i " aldagaiak "j" indibiduoan hartzen duen balioa da.

X_1 aldagai bakoitzen kalkula dezakegu:

- Aurkezpenean duen media aritmetikoa

$$\bar{x}_i = \frac{1}{N} \sum_{j=1}^N x_i^j \quad \forall i = 1, \dots, p$$

- Aurkezpenean duen bariantza, sakabanatze neurri bat bezala.

$$S_{x_i}^2 = \frac{1}{N} \sum_{j=1}^N (x_i^j - \bar{x}_i)^2 \quad \forall i = 1, \dots, p$$

x_1, x_2, \dots, x_p aldagai multzoan kalkula dezakegu, aldagaien arteko errelazio edo sinkronizazio neurri batzuk bezala:

- aldagai bikoteen kobariantzak:

$$\text{Kob}(x_i, x_N) = \frac{1}{N} \sum_{j=1}^N (x_i^j - \bar{x}_i) (x_N^j - \bar{x}_N) \quad \forall i \neq k \quad i, k = 1, \dots, p$$

$$\text{Kob}(x_i, x_i) = \frac{1}{N} \sum_{j=1}^N (x_i^j - \bar{x}_i)^2 = S_{x_i} \quad \forall i = 1, \dots, p$$

- aldagai bikoteen korrelazioak:

$$r(x_i, x_N) = \frac{\text{Kob}(x_i, x_N)}{S_{x_i} \cdot S_{x_N}} = \frac{\sum (x_i^j - \bar{x}_i)(x_N^j - \bar{x}_N)}{[\sum (x_i^j - \bar{x}_i)]^{1/2} \cdot [\sum (x_N^j - \bar{x}_N)]^{1/2}} \quad \forall i \neq k \\ i, k = 1, \dots, p$$

$$r(x_i, x_i) = \frac{\text{Kob}(x_i, x_i)}{S_{x_i} \cdot S_{x_i}} = \frac{S_{x_i}^2}{S_{x_i}^2} = 1 \quad \forall i = 1, \dots, p$$

II. TAULAREN LEHEN TRANSFORMAZIOAK

(A) TAULA ZENTRATUA

x_1, x_2, \dots, x_p aldagaiak unitate berdinetan neurtuak badira, taula zentratu batetan transformatu dugu, non x_i balioak beren media aritmetikotik neurtzen ditugu.

Transformazio honetan ez dugu informazio estadistikorik galtzen.

	1	J	N
$x_1 \rightarrow$	x_1^1	$x_1^j - x_1$	$x_1^N - x_1$
\vdots	\vdots	\vdots	\vdots
$x_i \rightarrow$	x_i^1	$x_i^j - x_i$	$x_i^N - x_i$
\vdots	\vdots	\vdots	\vdots
$x_p \rightarrow$	x_p^1	$x_p^j - x_p$	$x_p^N - x_p$

"TAULA ZENTRATUA"

$$x_i^1 = x_i - \bar{x}_i \quad \forall i = 1, \dots, p$$

$$\bar{x}_i^1 = \frac{1}{N} \sum_j (x_i^j - \bar{x}_i) = 0 \quad \forall i = 1, \dots, p$$

(B) TAULA TIPIFIKATUA

x_1, x_2, \dots, x_p aldagaiak unitate desberdinetan neurtuak badira, hasierako taula, taula tipifikatu batetan transformatu dugu (zentratua eta standarizatua)

Transformazio honetan informazio estadistikoa galduarren (aldagai bakoitzaren sakabanakuntza neurria galtzen baitugu, indibiduen arteko sakabanakuntza errelatiboa gordetzen badugu ere), unitate desberdinak eraginik ez dute.

	1	J	N
$x_1 \rightarrow$	$Z_1^1 = \frac{x_1^1 - \bar{x}_1}{S_{x_1}}$	$Z_1^j = \frac{x_1^j - \bar{x}_1}{S_{x_1}}$	$Z_1^N = \frac{x_1^N - \bar{x}_1}{S_{x_1}}$
\vdots	\vdots	\vdots	\vdots
$x_i \rightarrow$	$Z_i^1 = \frac{x_i^1 - \bar{x}_i}{S_{x_i}}$	$Z_i^j = \frac{x_i^j - \bar{x}_i}{S_{x_i}}$	$Z_i^N = \frac{x_i^N - \bar{x}_i}{S_{x_i}}$
\vdots	\vdots	\vdots	\vdots
$x_p \rightarrow$	$Z_p^1 = \frac{x_p^1 - \bar{x}_p}{S_{x_p}}$	$Z_p^j = \frac{x_p^j - \bar{x}_p}{S_{x_p}}$	$Z_p^N = \frac{x_p^N - \bar{x}_p}{S_{x_p}}$

"TAULA TIPIFIKATUA"

$$Z_i^1 = \frac{x_i^1 - \bar{x}_i}{S_{x_i}} \quad \forall i = 1, \dots, p$$

$$\bar{Z}_i = 0 \quad \forall i = 1, \dots, p; \quad s_{Z_i}^2 = 1 \quad \forall i = 1, \dots, p; \quad \text{Kob}(Z_i, Z_N) = r(Z_i, Z_N) \quad \forall i \neq k$$

2. TAULAREN ADIERAZPIDEAK

2.1. MATRITZEKO ADIERAZPIDEA:

(A) - Taula zentratu baten matritzezko adierazpidea:

$$X_{p \cdot N} = \begin{bmatrix} x_1^1 & x_1^2 & \dots & x_1^j & \dots & x_1^N \\ \vdots & \vdots & & \vdots & & \vdots \\ x_i^1 & x_i^2 & \dots & x_i^j & \dots & x_i^N \\ \vdots & \vdots & & \vdots & & \vdots \\ x_p^1 & x_p^2 & \dots & x_p^j & \dots & x_p^N \end{bmatrix} \quad \text{Non: } x_i^j = x_i^j - \bar{x}_i$$

- Media aritmetikoen zutabe matritza:

$$\bar{x}_{p \cdot 1} = \begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \\ \vdots \\ \bar{x}_i \\ \vdots \\ \bar{x}_p \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad \text{non: } \bar{x}_i = \frac{1}{N} \sum_j x_i^j \quad \forall i$$

- $x_1 \ x_2 \ \dots \ x_p$ -en kobariantza matritza:

$$K_{p \cdot p}^{x_1 - x_p} = \begin{bmatrix} S_{x_1}^2 & K(x_2 x_2) & \dots & K(x_1 x_p) \\ K(x_2 x_1) & S_{x_2}^2 & \dots & K(x_2 x_p) \\ \vdots & \vdots & \ddots & \vdots \\ K(x_p x_2) & K(x_p x_2) & \dots & S_{x_p}^2 \end{bmatrix}$$

 $K_{p \cdot p}$ matritza simetrikoa da:

$$\forall i \neq k \quad k(x_i x_N) = k(x_N x_i) = \frac{1}{N} \sum_j x_i^j \cdot x_N^j \quad \text{delako.}$$



- $x_1 x_2 \dots x_p$ -en korrelazio matritza:

$$R_{p,p}^{x_1 \dots x_p} = \begin{bmatrix} 1 & r(x_1 x_2) & \dots & r(x_1 x_p) \\ r(x_2 x_1) & 1 & \dots & r(x_2 x_p) \\ \vdots & \vdots & \ddots & \vdots \\ r(x_p x_1) & r(x_p x_2) & \dots & 1 \end{bmatrix}$$

$R_{p,p}$ matritza simetrikoa da:

$$\forall i \neq k \quad r(x_i x_k) = r(x_k x_i) = \frac{k(x_k x_i)}{s_{x_i} s_{x_k}} = \frac{\sum x_i^j \cdot x_k^j}{[\sum (x_i^j)^2]^{1/2} [\sum (x_k^j)^2]^{1/2}} \text{ delako.}$$

(B) - Taula tipifikatu baten matritzezko adierazpidea:

$$Z_{p,N} = \begin{bmatrix} Z_1^1, Z_1^2 & \dots & Z_1^j & \dots & Z_1^N \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ Z_i^1, Z_i^2 & \dots & Z_i^j & \dots & Z_i^N \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ Z_p^1, Z_p^2 & \dots & Z_p^j & \dots & Z_p^N \end{bmatrix} \quad \text{non: } Z_i^j = \frac{x_i^j - \bar{x}_i}{s_{x_i}}$$

- Media aritmetikoen matritze zutabea:

$$\bar{Z}_{p,1} = \begin{bmatrix} \bar{Z}_1 \\ \vdots \\ \bar{Z}_i \\ \vdots \\ \bar{Z}_p \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad \text{non: } \bar{Z}_i = \frac{1}{N} \sum_j Z_i^j \quad \forall i$$

- $Z_1 Z_2 \dots Z_p$ -en kobariantza matritza:

$$K_{p,p}^{Z_1 \dots Z_p} = \begin{bmatrix} 1 & k(Z_1 Z_2) & \dots & k(Z_1 Z_p) \\ k(Z_2 Z_1) & 1 & \dots & k(Z_2 Z_p) \\ \vdots & \vdots & \ddots & \vdots \\ k(Z_p Z_1) & k(Z_p Z_2) & \dots & 1 \end{bmatrix}$$

$K_{p,p}^{Z_1 \dots Z_p}$ matritze simetrikoa da: $k(Z_i Z_k) = \frac{1}{N} \sum_j Z_i^j \cdot Z_k^j = k(Z_k Z_i) \quad \forall i \neq k$
 eta $k(Z_i Z_i) = s_{Z_i}^2 = \frac{1}{N} \sum (Z_i^j)^2 = 1 \quad \forall i$

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2.2 GEOMETRIAZKO ADIERAZPIDEA

Taula bat emanik non:

	1	2	J	N
x_1	x_1^1	x_1^2	x_1^j	x_1^N
x_2	x_2^1	x_2^2	x_2^j	x_2^N
\vdots	\vdots	\vdots	\vdots	\vdots
x_i	x_i^1	x_i^2	x_i^j	x_i^N
\vdots	\vdots	\vdots	\vdots	\vdots
x_p	x_p^1	x_p^2	x_p^j	x_p^N

bere ikerketa askoz errezago bihurtzen zaigu, geometrikoki multidimensional espazio batetan adierazten badugu.

I. Gogora desagun: \mathbb{R}^N espazio bektorial euklidearra (distantzia euklideardun espazio bektorial metrikoa):

Definitzen badugu, \mathbb{R}^N (e.b.) -ean, eskalar biderkadura honela:

$$\langle \vec{X} \cdot \vec{Y} \rangle = (x_1 x_2 \dots x_N) \begin{pmatrix} 1 & & 0 \\ & 1 & \\ 0 & & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{pmatrix} = \sum_{j=1}^N x_j \cdot y_j$$

Orduan definizioz (ondorio bezala):

a) bektore baten euklidear norma:

$$\|\vec{X}\| = \langle \vec{X} \cdot \vec{X} \rangle^{1/2} = \left[(x_1 x_1 \dots x_N) \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{pmatrix} \right]^{1/2} = \left(\sum_{j=1}^N x_j^2 \right)^{1/2}$$

izango da.

b) bi bektoreen arteko euklidear distantzia:

$$d(\vec{X}_1 \vec{Y}) = \|\vec{X} - \vec{Y}\| = \left[\sum_{j=1}^N (x_j - y_j)^2 \right]^{1/2}$$

c) bi bektoreek eratzen duten angeluaren kosinua:

$$\cos \varphi(\vec{X}_1 \vec{Y}) = \frac{\langle \vec{X} \cdot \vec{Y} \rangle}{\|\vec{X}\| \cdot \|\vec{Y}\|} ; -1 \leq \cos \varphi(\vec{X}_1 \vec{Y}) \leq 1$$

$$\text{Hots: } \langle \vec{X} \cdot \vec{Y} \rangle = \|\vec{X}\| \cdot \|\vec{Y}\| \cdot \cos \varphi(\vec{X}_1 \vec{Y})$$

II. \mathbb{R}^N - (espazio bektorial euklidear)-ean, adierazten baditugu

x_1, x_2, \dots, x_p aldagaiak (suposatuko dugu guttienez zen tratuak direla):

$\vec{x}_1, \vec{x}_2, \dots, \vec{x}_p$ bektore bezala, non x_1 aldagai (zentsatu) bakoitzari,

$\mathbb{R}^N - n$

$\vec{x}_i = (x_i^1, x_i^2, \dots, x_i^j, \dots, x_i^N)$ bektorea dagokio.

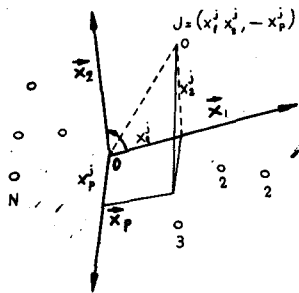
(\vec{x}_i bektorearen koordenadak, x_i aldagaiak indibiduo bakoitzean hartzen dituen balioak dira).

- $\langle \vec{x}_1, \vec{x}_2, \dots, \vec{x}_p / \vec{x}_i = (x_i^1, x_i^2, \dots, x_i^j, \dots, x_i^N) \rangle$ bektore multzoak azpiespazio euklidear bat eratzen dute \mathbb{R}^N espazio euklidearrean.

- eta $S = \langle 0, \vec{x}_1, \vec{x}_2, \dots, \vec{x}_p \rangle$ erreferentzial sistema bezela harturik, "J" indibiduo bakoitza, puntu batez adieraziko dugu.

(Non "J" puntuaren p-Koordenadak: $(x_1^j, x_2^j, \dots, x_p^j)$ aldagai guztiek "J" indibiduen hartzen dituzten balioak dira).

Hots: \mathbb{R}^N



III. Adierazpide honetan ikus dezakegu:

(1) \vec{X}_i, \vec{X}_k , bektoreen e. eskalar biderkadura, N bider $X_i X_k$ aldagaien

kobariantza dela:

$$\text{Hots: } \boxed{\langle \vec{X}_i \cdot \vec{X}_k \rangle = N \cdot \text{kob}(X_i X_k)} \quad \forall i, k$$

$$\langle \vec{X}_i \cdot \vec{X}_k \rangle = \sum_{j=1}^N X_i^j \cdot X_k^j$$

$$\text{Kob}(X_i X_k) = \frac{1}{N} \sum_{j=1}^N (X_i^{j-0})(X_k^{j-0}) \quad \left| \Rightarrow \langle \vec{X}_i \vec{X}_k \rangle = N \cdot \text{kob}(X_i X_k) \right.$$

f.n.c.b.

(2) \vec{X}_i bektorearen euklidear norma, \sqrt{N} bider X_i aldagaiaren

desbidazio standarra dela:

$$\text{Hots: } \boxed{\|\vec{X}_i\| = \sqrt{N} \cdot S_{X_i}} \quad \forall i$$

$$\|\vec{X}_i\| = \langle \vec{X}_i \vec{X}_i \rangle^{1/2} \stackrel{(1)}{=} [N \cdot \text{kob}(X_i X_i)]^{1/2} = [N \cdot S_{X_i}^2]^{1/2} = \sqrt{N} S_{X_i}$$

f.n.c.b.

(3) \vec{X}_i, \vec{X}_k bektoreak eratzen duten angeluen kosinua,

X_i, X_k aldagaien arteko korrelazio estadistikoa da

$$\text{Hots: } \boxed{\cos \varphi(\vec{X}_i \vec{X}_k) = r(X_i X_k)} \quad \forall i, k$$

$$\cos \varphi(\vec{X}_i, \vec{X}_k) = \frac{\langle \vec{X}_i \vec{X}_k \rangle}{\|\vec{X}_i\| \cdot \|\vec{X}_k\|} \stackrel{(1)}{=} \frac{N \cdot \text{kob}(X_i X_k)}{\sqrt{N} \cdot S_{X_i} \cdot \sqrt{N} S_{X_k}} \stackrel{(2)}{=} \frac{\text{kob}(X_i X_k)}{S_{X_i} S_{X_k}} = r(X_i X_k)$$

f.n.c.b.

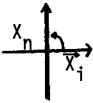
$$\boxed{-1 \leq \cos \varphi(\vec{X}_i \vec{X}_k) = r(X_i X_k) \leq 1} \quad \forall i, k$$



Orduan esan dezakegu:

● \vec{X}_1, \vec{X}_n bi bektore adierazleek eratzten duten angelua, zenbat eta txikiago den, dagozkion aldagaien korrelazioa hainbat handiagoa dela.

(a) X_i, X_n aldagaiak korrelaziogabeak izango dira, soilik eta baldin soilik, bere bektore adierazleak ortogonalak badira $R^N - n$



Hots: $r(X_i X_k) = 0 \iff \cos \varphi(\vec{X}_i, \vec{X}_k) = 0$ edo $\varphi(\vec{X}_i, \vec{X}_k) = 90^\circ$;
 $\vec{X}_i \perp \vec{X}_k \quad \langle \vec{X}_i, \vec{X}_k \rangle = 0$

(b) X_i, X_n aldagaien korrelazioa ± 1 izango da, soilik eta baldin soilik beren artean menpekotasun lineal bat badago.

Hots: $r(X_i X_k) = \pm 1 \iff \cos \varphi(\vec{X}_i, \vec{X}_k) = \pm 1$ edo $\varphi(\vec{X}_i, \vec{X}_k) = 0^\circ, 180^\circ$;



$\exists \lambda / \vec{X}_i = \vec{X}_k$

* Taulan existitzen bada, X_i aldagaien bat, non menpekotasun lineal bat du beste batekin X_n edo beste batzuekin X_h, X_k - orduan ikerketarako X_i aldagaia baztertu egingo dugu.

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IV. Orduan taulen adierazpide geometrikoa, bi kasuetan honela izango litzateke:

(A) X_1, X_2, \dots, X_p aldagaiek zentratuak baldin badira, hots:

$\bar{x}_1 = 0, \bar{x}_2 = 0, \dots, \bar{x}_p = 0$, lehenago esan dugunetz beti lortu genezake hau, eta gainera informazio estadistikorik galdu gabe.

Eta dagokion taula zentratua; bada:

	1	2	J	N
X_1	x_1^1	x_1^2	x_1^j	x_1^N
X_2	x_2^1	x_2^2	x_2^j	x_2^N
X_i	x_i^1	x_i^2	x_i^j	x_i^N
X_p	x_p^1	x_p^2	x_p^j	x_p^N

Orduan, adierazten badugu taula geometrikoki, R^N indibiduen espazio euklidearrean:

Eta baldin badira $\bar{x}_1, \bar{x}_2, \dots, \bar{x}_p, X_1, X_2, \dots, X_p$ aldagaiari dagozkien koordenatatako bektore adierazleak:

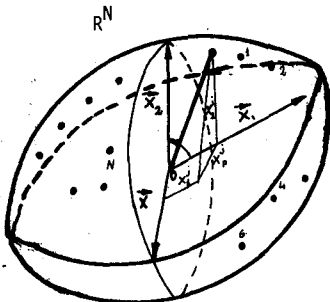
$$\bar{x}_1 = (x_1^1, x_1^2, \dots, x_1^N) \in R^N$$

$$\bar{x}_i = (x_i^1, x_i^2, \dots, x_i^N) \in R^N$$

$$\bar{x}_p = (x_p^1, x_p^2, \dots, x_p^N) \in R^N$$

$\bar{x}_1, \bar{x}_2, \dots, \bar{x}_p$ bektoreen muturrak, jatorrian zentratuta dagoen elipsoide batetan aurkitzen dira, non elipsoide honen p - ardatzak, aldagaien desbidazio standarrari proportzionalak dira, eta ez dute zergatik ortogonalak beren artean izan behar.

Hots:



$$\bar{x}_i = (x_i^1, x_i^2, \dots, x_i^N) \in R^N$$

$$s = \langle 0, \bar{x}_1, \bar{x}_2, \dots, \bar{x}_p \rangle$$

$$\|\bar{x}_i\| = \sqrt{N} \cdot s_{x_i} \quad \forall i = 1, \dots, p$$

$$\cos \varphi(\bar{x}_i, \bar{x}_k) = r(x_i, x_k) \quad \forall i, k$$

$$j = (x_1^j, \dots, x_p^j) \quad \forall j = 1, \dots, N$$

the study. The first author (SM) was the primary investigator and was responsible for the design, data collection, data analysis and writing of the manuscript. The second author (MM) was responsible for the design, data collection, data analysis and writing of the manuscript. The third author (MM) was responsible for the design, data collection, data analysis and writing of the manuscript. The fourth author (MM) was responsible for the design, data collection, data analysis and writing of the manuscript. The fifth author (MM) was responsible for the design, data collection, data analysis and writing of the manuscript. The sixth author (MM) was responsible for the design, data collection, data analysis and writing of the manuscript. The seventh author (MM) was responsible for the design, data collection, data analysis and writing of the manuscript. The eighth author (MM) was responsible for the design, data collection, data analysis and writing of the manuscript. The ninth author (MM) was responsible for the design, data collection, data analysis and writing of the manuscript. The tenth author (MM) was responsible for the design, data collection, data analysis and writing of the manuscript.

The study was approved by the ethics committee of the University of Toronto. All participants gave their informed consent before participating in the study. The study was conducted in a laboratory setting. Participants were recruited from a local community and were screened for any medical conditions that might affect their ability to perform the tasks. The study was conducted over a period of 12 weeks. Participants were trained for 4 weeks before the start of the study. The study was conducted in a laboratory setting. Participants were recruited from a local community and were screened for any medical conditions that might affect their ability to perform the tasks. The study was conducted over a period of 12 weeks. Participants were trained for 4 weeks before the start of the study.

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(B) X_1, X_2, \dots, X_p aldagaiak tipifikatzen baldin badira, hots:

$$Z_i = \frac{X_i - \bar{X}_i}{S_{X_i}} \quad \forall i \quad ; \quad \bar{Z}_1 = 0, \bar{Z}_2 = 0, \dots, \bar{Z}_p = 0 \quad \text{eta} \quad S_{Z_1}^2 = 1, S_{Z_2}^2 = 1, \dots, S_{Z_p}^2 = 1$$

Dakigunez aldagaiek tipifikatzean informazioa galtzen dugu, baina honela aldagaiek neurtuta duten unitate desberdinak ez dute eraginik. Dagokion - taula tipifikatua bada:

	1	2	...	J	...	N
$X_1 \rightarrow Z_1$	Z_1^1	Z_1^2	...	Z_1^j	...	Z_1^N
$X_2 \rightarrow Z_2$	Z_2^1	Z_2^2	...	Z_2^j	...	Z_2^N
$X_i \rightarrow Z_i$	Z_i^1	Z_i^2	...	Z_i^j	...	Z_i^N
$X_p \rightarrow Z_p$	Z_p^1	Z_p^2	...	Z_p^j	...	Z_p^N

Orduan, adierazten badugu taula, geometrikoki, R^N indibiduen espazio euklidearrean:

Eta baldin badira $\bar{Z}_1, \bar{Z}_2, \dots, \bar{Z}_p, Z_1, Z_2, \dots, Z_p$ aldagai tipifikatuai dagozkien bektore adierazleak:

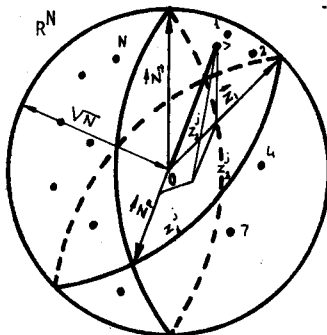
$$\bar{Z}_1 = (Z_1^1, Z_1^2, \dots, Z_1^N) \in R^N$$

$$\bar{Z}_i = (Z_i^1, Z_i^2, \dots, Z_i^N) \in R^N$$

$$\bar{Z}_p = (Z_p^1, Z_p^2, \dots, Z_p^N) \in R^N$$

$\bar{Z}_1, \bar{Z}_2, \dots, \bar{Z}_p$ bektoreen muturrak, jatorrian zentratuta dagoen esfera baten aurkitzen dira, non esfera honen arradioa \sqrt{N} izango da.

Hots:



$$\bar{Z}_i = (Z_i^1, Z_i^2, \dots, Z_i^N) \in R^N$$

$$s = \langle 0, \bar{Z}_1, \bar{Z}_2, \dots, \bar{Z}_p \rangle$$

$$\|\bar{Z}_i\|^2 = \sqrt{N} \cdot S_{Z_i} = \sqrt{N} \cdot 1 \quad \forall i=1, \dots, p$$

$$\cos \varphi(\bar{Z}_i, \bar{Z}_k) = r(Z_i, Z_k) = k(Z_i, Z_k) = N(\bar{Z}_i, \bar{Z}_k)$$

$$j = (Z_i^j, Z_2^j, \dots, Z_p^j) \quad \forall j=1, \dots, N$$

the \mathbb{R}^n -valued function \mathbf{f} is a solution of the system (1) if and only if

$$\mathbf{f}'(x) = \mathbf{A}(x)\mathbf{f}(x) + \mathbf{b}(x) \quad (2)$$

where $\mathbf{A}(x) = (a_{ij}(x))$ is an $n \times n$ matrix and $\mathbf{b}(x) = (b_1(x), \dots, b_n(x))$ is an n -vector, both of them continuous on I .

Let us assume that $\mathbf{A}(x)$ and $\mathbf{b}(x)$ are continuous on I and that $\mathbf{A}(x)$ is invertible for all $x \in I$.

Let us denote by $\mathbf{f}_1(x), \dots, \mathbf{f}_n(x)$ the solutions of the homogeneous system

$$\mathbf{f}'(x) = \mathbf{A}(x)\mathbf{f}(x) \quad (3)$$

and by $\mathbf{f}_0(x)$ the particular solution of the inhomogeneous system (2).

Let us denote by $\mathbf{f}(x)$ the general solution of the inhomogeneous system (2).

Let us assume that $\mathbf{f}(x)$ is a solution of the system (2) and that $\mathbf{f}(x)$ is a solution of the system (3).

Let us denote by $\mathbf{f}_1(x), \dots, \mathbf{f}_n(x)$ the solutions of the homogeneous system (3).

Let us denote by $\mathbf{f}_0(x)$ the particular solution of the inhomogeneous system (2).

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Let us denote by $\mathbf{f}_1(x), \dots, \mathbf{f}_n(x)$ the solutions of the homogeneous system (3).

Let us denote by $\mathbf{f}_0(x)$ the particular solution of the inhomogeneous system (2).

Let us denote by $\mathbf{f}(x)$ the general solution of the inhomogeneous system (2).

3. OSAGAI NAGUSIEN ANALISIA

Suposa dezagun egin ditugula p-test $X_1 X_2 \dots X_p$ edo p- aldagaien azterketa N-indibiduei buruz.

Eta erresultatuen "taula" hau dugula:

	1	2	J	N
X_1	X_1^1	X_1^2	X_1^j	X_1^N
X_2	X_2^1	X_2^2	X_2^j	X_2^N
X_p	X_p^1	X_p^2	X_p^j	X_p^N

I. Zer lortu genezake osagai nagusien analisitaz?

Azterketako p-testak $X_1 X_2 \dots X_p$, non gehienetan beren artean alkar korrelazionaturik daude, beste r-testara ($r < p$) erreduzitu, bitez $Y_1 Y_2 \dots Y_r$ non hоек ez dira alkar korrelazionatuak, eta sakabanakuntza guztiaren % asko esplikatzen dute.

Gero azterketa bakoitzean, "Osagai nagusiak", deituko ditugun " $Y_1 Y_2 \dots Y_r$ " aldagai berri hoiei, esanahi bat emango diogu, haserako $X_1 X_2 \dots X_p$ aldagaien esanahiarekin loturik noski.

Eta osagai nagusietan gure N-indibiduoek hartzen dituzten balioak taula batetan azalduz; hots:

	1	2	J	N
Y_1	Y_1^1	Y_1^2	Y_1^j	Y_1^N
Y_2	Y_2^1	Y_2^2	Y_2^j	Y_2^N
Y_r	Y_r^1	Y_r^2	Y_r^j	Y_r^N

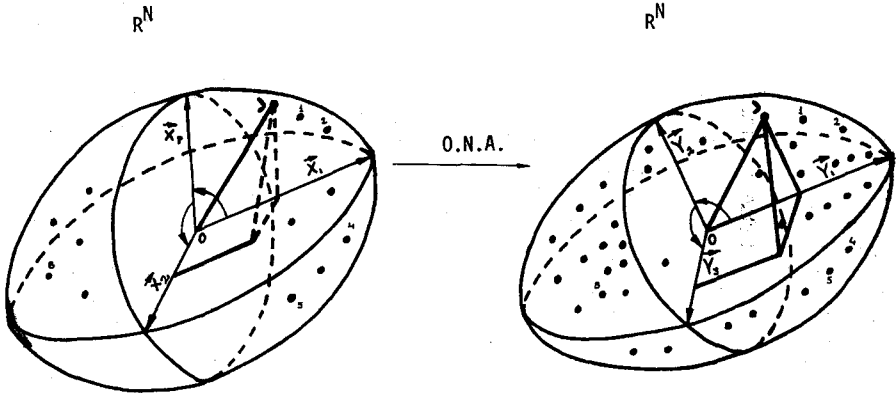
$$r(Y_i Y_k) = 0 \quad \forall i \neq k$$

$$i, k = 1, \dots, r$$

$$r < p$$

N-indibiduoak aztertuko ditugu.

II. PLANTEIAMENDU GEOMETRIKOA



$$S = \langle 0, \bar{x}_1, \bar{x}_2, \dots, \bar{x}_p \rangle$$

$$J = (x_1^j, x_2^j, \dots, x_p^j) \quad \forall J = 1, \dots, N$$

$$S' = \langle 0, \bar{y}_1, \bar{y}_2, \dots, \bar{y}_p \rangle$$

$$\text{non: } \left\{ \begin{array}{l} \text{a) } \bar{y}_1 \perp \bar{y}_2 \perp \dots \perp \bar{y}_p \\ \text{b) } \|\bar{y}_1\| > \|\bar{y}_2\| > \dots > \|\bar{y}_p\| \end{array} \right.$$

edo

$$D_{y_1}^2 \leq D_{y_2}^2 \leq \dots \leq D_{y_p}^2 \quad /$$

$$D_{y_i}^2 = \sum_j y_i^j \quad / D_{y_i}^2 = \sum_j y_i^j \quad \forall i = 1, \dots, p$$

$$J = (y_1^j, y_2^j, \dots, y_p^j) \quad \forall J = 1, \dots, N$$

Geometrikoki adierazten baditugu N -indibiduoak, N puntutaz, $S = \langle 0, \bar{x}_1, \bar{x}_2, \dots, \bar{x}_p \rangle$ sisteman non ardatzak ez dute zergaitik ortogonalak izan behar beren artean, orduan "osagai nagusien analisiak" transformatzan du S sistema, beste jatorri berdina duen $S' = \langle 0, \bar{y}_1, \bar{y}_2, \dots, \bar{y}_p \rangle$ sisteman, non " $\bar{y}_1, \bar{y}_2, \dots, \bar{y}_p$ ": osagai nagusien bektore adierazleak izango dira, eta ondoko propietateak beteko dituzte:

- $\bar{y}_1, \bar{y}_2, \dots, \bar{y}_p$, jatorrian zentratua dagoen, elipsoide baten ardatz ortogonalak dira, garrantzi beherakoiak ordenaturik edo $-\bar{y}_1, \bar{y}_2, \dots, \bar{y}_p$, jatorrian pasatzen diran, ingurakoi-zuzen sorta ortogonal bat da.

1. $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$
 2. $\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$
 3. $\frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$
 4. $\frac{1}{4} \times \frac{1}{8} = \frac{1}{32}$
 5. $\frac{1}{8} \times \frac{1}{8} = \frac{1}{64}$
 6. $\frac{1}{8} \times \frac{1}{16} = \frac{1}{128}$
 7. $\frac{1}{16} \times \frac{1}{16} = \frac{1}{256}$
 8. $\frac{1}{16} \times \frac{1}{32} = \frac{1}{512}$
 9. $\frac{1}{32} \times \frac{1}{32} = \frac{1}{1024}$
 10. $\frac{1}{32} \times \frac{1}{64} = \frac{1}{2048}$
 11. $\frac{1}{64} \times \frac{1}{64} = \frac{1}{4096}$
 12. $\frac{1}{64} \times \frac{1}{128} = \frac{1}{8192}$
 13. $\frac{1}{128} \times \frac{1}{128} = \frac{1}{16384}$
 14. $\frac{1}{128} \times \frac{1}{256} = \frac{1}{32768}$
 15. $\frac{1}{256} \times \frac{1}{256} = \frac{1}{65536}$
 16. $\frac{1}{256} \times \frac{1}{512} = \frac{1}{131072}$
 17. $\frac{1}{512} \times \frac{1}{512} = \frac{1}{262144}$
 18. $\frac{1}{512} \times \frac{1}{1024} = \frac{1}{524288}$
 19. $\frac{1}{1024} \times \frac{1}{1024} = \frac{1}{1048576}$
 20. $\frac{1}{1024} \times \frac{1}{2048} = \frac{1}{2097152}$
 21. $\frac{1}{2048} \times \frac{1}{2048} = \frac{1}{4194304}$
 22. $\frac{1}{2048} \times \frac{1}{4096} = \frac{1}{8388608}$
 23. $\frac{1}{4096} \times \frac{1}{4096} = \frac{1}{16777216}$
 24. $\frac{1}{4096} \times \frac{1}{8192} = \frac{1}{33554432}$
 25. $\frac{1}{8192} \times \frac{1}{8192} = \frac{1}{67108864}$
 26. $\frac{1}{8192} \times \frac{1}{16384} = \frac{1}{134217728}$
 27. $\frac{1}{16384} \times \frac{1}{16384} = \frac{1}{268435456}$
 28. $\frac{1}{16384} \times \frac{1}{32768} = \frac{1}{536870912}$
 29. $\frac{1}{32768} \times \frac{1}{32768} = \frac{1}{1073741824}$
 30. $\frac{1}{32768} \times \frac{1}{65536} = \frac{1}{2147483648}$
 31. $\frac{1}{65536} \times \frac{1}{65536} = \frac{1}{4294967296}$
 32. $\frac{1}{65536} \times \frac{1}{131072} = \frac{1}{8589934592}$
 33. $\frac{1}{131072} \times \frac{1}{131072} = \frac{1}{17179869184}$
 34. $\frac{1}{131072} \times \frac{1}{262144} = \frac{1}{34359738368}$
 35. $\frac{1}{262144} \times \frac{1}{262144} = \frac{1}{68719476736}$
 36. $\frac{1}{262144} \times \frac{1}{524288} = \frac{1}{137438953472}$
 37. $\frac{1}{524288} \times \frac{1}{524288} = \frac{1}{274877906944}$
 38. $\frac{1}{524288} \times \frac{1}{1048576} = \frac{1}{549755813888}$
 39. $\frac{1}{1048576} \times \frac{1}{1048576} = \frac{1}{1099511627776}$
 40. $\frac{1}{1048576} \times \frac{1}{2147483648} = \frac{1}{2199023255552}$
 41. $\frac{1}{2147483648} \times \frac{1}{2147483648} = \frac{1}{4398046511104}$
 42. $\frac{1}{2147483648} \times \frac{1}{4398046511104} = \frac{1}{8796093022208}$
 43. $\frac{1}{4398046511104} \times \frac{1}{4398046511104} = \frac{1}{17592186044416}$
 44. $\frac{1}{4398046511104} \times \frac{1}{8796093022208} = \frac{1}{35184372088832}$
 45. $\frac{1}{8796093022208} \times \frac{1}{8796093022208} = \frac{1}{70368744177664}$
 46. $\frac{1}{8796093022208} \times \frac{1}{14061456035312} = \frac{1}{124627008315008}$
 47. $\frac{1}{14061456035312} \times \frac{1}{14061456035312} = \frac{1}{196858176439232}$
 48. $\frac{1}{14061456035312} \times \frac{1}{29122912070624} = \frac{1}{393716352878464}$
 49. $\frac{1}{29122912070624} \times \frac{1}{29122912070624} = \frac{1}{787432705756928}$
 50. $\frac{1}{29122912070624} \times \frac{1}{58245824141248} = \frac{1}{1574865411513856}$
 51. $\frac{1}{58245824141248} \times \frac{1}{58245824141248} = \frac{1}{3149730823027712}$
 52. $\frac{1}{58245824141248} \times \frac{1}{116491648282496} = \frac{1}{6299461646055424}$
 53. $\frac{1}{116491648282496} \times \frac{1}{116491648282496} = \frac{1}{12598923292110848}$
 54. $\frac{1}{116491648282496} \times \frac{1}{231983265564992} = \frac{1}{25197846584221696}$
 55. $\frac{1}{231983265564992} \times \frac{1}{231983265564992} = \frac{1}{50395693168443392}$
 56. $\frac{1}{231983265564992} \times \frac{1}{463963861129984} = \frac{1}{100791386336886784}$
 57. $\frac{1}{463963861129984} \times \frac{1}{463963861129984} = \frac{1}{201582772673773568}$
 58. $\frac{1}{463963861129984} \times \frac{1}{927927722259968} = \frac{1}{403165545347547136}$
 59. $\frac{1}{927927722259968} \times \frac{1}{927927722259968} = \frac{1}{806331090695094272}$
 60. $\frac{1}{927927722259968} \times \frac{1}{1854654444519936} = \frac{1}{1612662181390188544}$
 61. $\frac{1}{1854654444519936} \times \frac{1}{1854654444519936} = \frac{1}{3225324362780377088}$
 62. $\frac{1}{1854654444519936} \times \frac{1}{3648908889039872} = \frac{1}{6450648725560754176}$
 63. $\frac{1}{3648908889039872} \times \frac{1}{3648908889039872} = \frac{1}{12901297451121508352}$
 64. $\frac{1}{3648908889039872} \times \frac{1}{7297817778079744} = \frac{1}{25802594902243016704}$
 65. $\frac{1}{7297817778079744} \times \frac{1}{7297817778079744} = \frac{1}{51605189804486033408}$
 66. $\frac{1}{7297817778079744} \times \frac{1}{14595635556159488} = \frac{1}{103210379608972066816}$
 67. $\frac{1}{14595635556159488} \times \frac{1}{14595635556159488} = \frac{1}{206420759217944133632}$
 68. $\frac{1}{14595635556159488} \times \frac{1}{29191271112318976} = \frac{1}{412841518435888267264}$
 69. $\frac{1}{29191271112318976} \times \frac{1}{29191271112318976} = \frac{1}{825683036871776534528}$
 70. $\frac{1}{29191271112318976} \times \frac{1}{58382542224637952} = \frac{1}{1651366073743553069056}$
 71. $\frac{1}{58382542224637952} \times \frac{1}{58382542224637952} = \frac{1}{3302732147487106138112}$
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 73. $\frac{1}{116765084449275904} \times \frac{1}{116765084449275904} = \frac{1}{13210928589848424552448}$
 74. $\frac{1}{116765084449275904} \times \frac{1}{233430168898551808} = \frac{1}{26421857179696849104896}$
 75. $\frac{1}{233430168898551808} \times \frac{1}{233430168898551808} = \frac{1}{52843714359393698209792}$
 76. $\frac{1}{233430168898551808} \times \frac{1}{466860337797103616} = \frac{1}{105687428718787396419584}$
 77. $\frac{1}{466860337797103616} \times \frac{1}{466860337797103616} = \frac{1}{211374857437574792839168}$
 78. $\frac{1}{466860337797103616} \times \frac{1}{933720675594207232} = \frac{1}{422749714875149585678336}$
 79. $\frac{1}{933720675594207232} \times \frac{1}{933720675594207232} = \frac{1}{845499429750299171356672}$
 80. $\frac{1}{933720675594207232} \times \frac{1}{1867441351188414464} = \frac{1}{1690998858500598342713344}$
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 82. $\frac{1}{1867441351188414464} \times \frac{1}{3734882702376828928} = \frac{1}{6763995434002393370853376}$
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 85. $\frac{1}{7469765404753657856} \times \frac{1}{7469765404753657856} = \frac{1}{54111963472019147166827008}$
 86. $\frac{1}{7469765404753657856} \times \frac{1}{14919530809507315712} = \frac{1}{108223926944038294333654016}$
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 91. $\frac{1}{59678123238029262848} \times \frac{1}{59678123238029262848} = \frac{1}{3463165662209225418676928512}$
 92. $\frac{1}{59678123238029262848} \times \frac{1}{119356246476058525696} = \frac{1}{6926331324418450837353857024}$
 93. $\frac{1}{119356246476058525696} \times \frac{1}{119356246476058525696} = \frac{1}{13852662695236905174707714048}$
 94. $\frac{1}{119356246476058525696} \times \frac{1}{238712492952117051392} = \frac{1}{27705325390473810349415428096}$
 95. $\frac{1}{238712492952117051392} \times \frac{1}{238712492952117051392} = \frac{1}{55410650780847620698830856192}$
 96. $\frac{1}{238712492952117051392} \times \frac{1}{477424985804234102784} = \frac{1}{110821301561695241397661712384}$
 97. $\frac{1}{477424985804234102784} \times \frac{1}{477424985804234102784} = \frac{1}{221642603123390482795323424768}$
 98. $\frac{1}{477424985804234102784} \times \frac{1}{954849971608468205568} = \frac{1}{443285206246780965590646849536}$
 99. $\frac{1}{954849971608468205568} \times \frac{1}{954849971608468205568} = \frac{1}{886570412493561731181293699072}$
 100. $\frac{1}{954849971608468205568} \times \frac{1}{1909699943216936411136} = \frac{1}{1773140824987123462362587398144}$
 101. $\frac{1}{1909699943216936411136} \times \frac{1}{1909699943216936411136} = \frac{1}{3546281649974246824725174796288}$
 102. $\frac{1}{1909699943216936411136} \times \frac{1}{3819399886433872822272} = \frac{1}{7092563299948493649450349592576}$
 103. $\frac{1}{3819399886433872822272} \times \frac{1}{3819399886433872822272} = \frac{1}{14185126599896987298900699185152}$
 104. $\frac{1}{3819399886433872822272} \times \frac{1}{7638799772867745644544} = \frac{1}{28370253199793974597801398370304}$
 105. $\frac{1}{7638799772867745644544} \times \frac{1}{7638799772867745644544} = \frac{1}{56740506399587949195602796740608}$
 106. $\frac{1}{7638799772867745644544} \times \frac{1}{15277599545735491289088} = \frac{1}{113481012799175898391205593481216}$
 107. $\frac{1}{15277599545735491289088} \times \frac{1}{15277599545735491289088} = \frac{1}{226962025598351795782411186962432}$
 108. $\frac{1}{15277599545735491289088} \times \frac{1}{31155197091470982576576} = \frac{1}{453924051196703591564822373924864}$
 109. $\frac{1}{31155197091470982576576} \times \frac{1}{31155197091470982576576} = \frac{1}{907848102393419651529644747849728}$
 110. $\frac{1}{31155197091470982576576} \times \frac{1}{62370394182843965152953} = \frac{1}{1815696204786839303059299495699456}$
 111. $\frac{1}{62370394182843965152953} \times \frac{1}{62370394182843965152953} = \frac{1}{3631392409573678606118598991398912}$
 112. $\frac{1}{62370394182843965152953} \times \frac{1}{124740788365687930305906} = \frac{1}{7262784819147357212237197982797824}$
 113. $\frac{1}{124740788365687930305906} \times \frac{1}{124740788365687930305906} = \frac{1}{14525567673134786064474385965595648}$
 114. $\frac{1}{124740788365687930305906} \times \frac{1}{249501576731375821209812} = \frac{1}{29051135346269572120948771931191296}$
 115. $\frac{1}{249501576731375821209812} \times \frac{1}{249501576731375821209812} = \frac{1}{58102270692539164241937543862382592}$
 116. $\frac{1}{249501576731375821209812} \times \frac{1}{499003153462751642438624} = \frac{1}{116204541285078328483875087724765184}$
 117. $\frac{1}{499003153462751642438624} \times \frac{1}{499003153462751642438624} = \frac{1}{232409082570156656967750175449530368}$
 118. $\frac{1}{499003153462751642438624} \times \frac{1}{998006306925503284877248} = \frac{1}{464818165140313313935501350899060736}$
 119. $\frac{1}{998006306925503284877248} \times \frac{1}{998006306925503284877248} = \frac{1}{929636330280626629771002701798121472}$
 120. $\frac{1}{998006306925503284877248} \times \frac{1}{1996012613851006459752496} = \frac{1}{1859272660561253259542005403596242944}$
 121. $\frac{1}{1996012613851006459752496} \times \frac{1}{1996012613851006459752496} = \frac{1}{3718545321122512919084010807192485888}$
 122. $\frac{1}{1996012613851006459752496} \times \frac{1}{3992025227702012919504992} = \frac{1}{7437090642245025839168021614384971776}$
 123. $\frac{1}{3992025227702012919504992} \times \frac{1}{3992025227702012919504992} = \frac{1}{14874181284490051738336043228769943552}$
 124. $\frac{1}{3992025227702012919504992} \times \frac{1}{7984050455404025839009984} = \frac{1}{29748362568980103476672086457539887104}$
 125. $\frac{1}{7984050455404025839009984} \times \frac{1}{7984050455404025839009984} = \frac{1}{59496725137960206793344172915079774208}$
 126. $\frac{1}{7984050455404025839009984} \times \frac{1}{15968100908808041678678968} = \frac{1}{118993450275920413583568345830159548416}$
 127. $\frac{1}{15968100908808041678678968} \times \frac{1}{15968100908808041678678968} = \frac{1}{237986908551840827173577691660319096832}$
 128. $\frac{1}{15968100908808041678678968} \times \frac{1}{31936201817616083356717936} = \frac{1}{475973817103681657147158723320638193664}$
 129. $\frac{1}{31936201817616083356717936} \times \frac{1}{31936201817616083356717936} = \frac{1}{951947634207363267134317446641276387328}$
 130. $\frac{1}{31936201817616083356717936} \times \frac{1}{63972403635232166713435872} = \frac{1}{1903895268414726534268636893282552774656}$
 131. $\frac{1}{63972403635232166713435872} \times \frac{1}{63972403635232166713435872} = \frac{1}{3807790536829453138537273786565105549312}$
 132. $\frac{1}{63972403635232166713435872} \times \frac{1}{127944807270464333426871744} = \frac{1}{7615581073658906277074547573130211098624}$
 133. $\frac{1}{127944807270464333426871744} \times \frac{1}{127944807270464333426871744} = \frac{1}{15231162147317866684149095146260422197248}$
 134. $\frac{1}{127944807270464333426871744} \times \frac{1}{255809614540928666853783488} = \frac{1}{30462324294635733370716790292520844394496}$
 135. $\frac{1}{255809614540928666853783488} \times \frac{1}{255809614540928666853783488} = \frac{1}{60924648589271466771433580585041688788992}$
 136. $\frac{1}{255809614540928666853783488} \times \frac{1}{511619229081857333707566976} = \frac{1}{121849297178542933542867371170083377577984}$
 137. $\frac{1}{511619229081857333707566976} \times \frac{1}{511619229081857333707566976} = \frac{1}{243698594357085867085734742340166755155968}$
 138. $\frac{1}{511619229081857333707566976} \times \frac{1}{1023238458163714667415133952} = \frac{1}{487397188714171734171468484680333510311936}$
 139. $\frac{1}{1023238458163714667415133952} \times \frac{1}{1023238458163714667415133952} = \frac{1}{974794377428343468422926969360667020623872}$
 140. $\frac{1}{1023238458163714667415133952} \times \frac{1}{2046476916327429334830267936} = \frac{1}{1949588754856686936845853878721334041247744}$
 141. $\frac{1}{2046476916327429334830267936} \times \frac{1}{2046476916327429334830267936} = \frac{1}{3899177431713373669671715757442668082495488}$
 142. $\frac{1}{2046476916327429334830267936} \times \frac{1}{4092953832654858669643435712} = \frac{1}{77983548634267473392868714148853361649$

Hots:

Lehenengo osagai nagusien bektore adierazlea: \vec{V}_1 jatorritikan pasatuaz, indibiduoek gehien inguratzen dituen zuzena izango da.

Hots: $\sum_{j=1}^N D^2(J, \vec{V}) = D_y^2$ minimatzen duena.

Ala baliokide dena:

Jatorritikan pasatuaz, sakabanakuntza edo bariantza gehien jasotzen duen zuzena, norma euklidearra gehien duena.

Hots: $S_y^2 \approx \|\vec{V}\|^2$ maximatzen duena.

Bigarren osagai nagusien bektore adierazlea: \vec{V}_2 jatorritikan pasatuaz, eta \vec{V}_1 -ri ortogonal izanik, indibiduoek gehien inguratzen dituen zuzena izango da, edo ortogonaletatik norma euklidear gehien duena edo sakabanakuntza (bariantza) gehien jasotzen duena.

P-garreno osagai nagusiaren bektore adierazlea: \vec{V}_p jatorritikan pasatuaz, eta $\vec{V}_1, \vec{V}_2, \dots, \vec{V}_{p-1}$ -ri ortogonal izanik, indibiduoek gehien inguratzen dituen zuzena izango da, edo ortogonaletatik norma euklidear

OSAGARRIAK

a) Zergatikan jatorritikan pasa arazten ditugu $\vec{V}_1, \vec{V}_2, \dots, \vec{V}_p$? Honela metrika S, ta S' -n berdina delako, euklidearra geure kasuan.

b) Zergatikan $\vec{V}_1 \perp \vec{V}_2 \perp \dots \perp \vec{V}_p$? Honela zuzen batek hartzen ez duen sakabanakuntza, besteek jasotzen duelako.

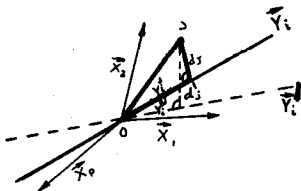
$$\|\vec{J}\|^2 = (Y_1^j)^2 + (Y_2^j)^2 + \dots + (Y_p^j)^2 = \sum_{j=1}^N (Y_i^j)^2 \quad (\text{Pitagorasen Teoremataz})$$

c) Euklidear espazio batetan, ta \vec{V}_1 jatorritik pasatzen bada, baliokideak dira:

<p>(a) $D_{y_i}^2 = \sum_{j=1}^N d^2(J, \vec{V}_i)$ minimizatu</p> <p>edo</p> <p>(a') $\ \vec{V}_i\ ^2 = N \cdot S_{y_i}^2$ maximalizatu</p>
--

Frogaketa:

(Pitagorasez teoremaz)



$$\|J\|^2 = d_j^2 + (Y_i^j)^2 = d_j'^2 + (Y_i'^j)^2 \quad \forall J$$

orduan edozein jatorritikan pasatzen diren

\bar{Y}_i, \bar{Y}_i' bi zuzenentzat; $d_j < d_j' \Rightarrow Y_i^j > Y_i'^j$

eta baliokidea da d_j minimizatu ala Y_i^j maximalizatu.

J-n gehiketa eginik:

$$\sum \|J\|^2 = \sum_j d^2(J, \bar{Y}_i) + \sum_j (Y_i^j)^2 = \sum_j d^2(J, \bar{Y}_i) + \|\bar{Y}_i\|^2 = Kte$$

orduan jatorritikan pasatzen diren \bar{Y}_i zuzenentzat, baliokidea da:

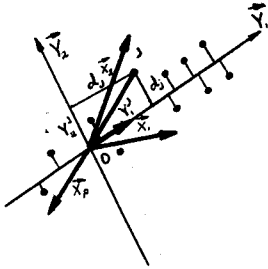
$$\sum_j d^2(J, \bar{Y}_i) = D_{y_i}^2 \quad \text{minimizatu}$$

ala

$$\sum_j (Y_i^j)^2 = \|\bar{Y}_i\|^2 = N S_{y_i}^2 \quad \text{maximalizatu} \quad \text{j.n.g.b.}$$

eta $\|\bar{Y}_i\|^2$ maximalizatzea, $\|\bar{Y}_i\|$ maximalizatzea da.

III. Ikus dezagun Osagai nagusiek aurkitzeko metodoa:



Planteamenduan esan dugunez, \vec{Y}_1 lehenengo osagai nagusiaren adierazleak, ondoko bi propietate bete behar ditu:

1.- Jatorritikan pasa:

$$\vec{Y}_1 = L_{11}\vec{X}_1 + L_{12}\vec{X}_2 + \dots + L_{1p}\vec{X}_p \text{ izango da}$$

non: $(-11 \ L_{12} \ \dots \ 1p)$, \vec{Y}_1 -n bertsore direktzionala da

$$= \sum_{i=1}^p l_{i1}^2 \quad ; \quad \cos \varphi(\vec{Y}_1, \vec{X}_k) = r(Y_1, X_k)$$

2.- $\sum_j d^2(J_1, \vec{Y}_1)$ minimo da, edo

$$S_{Y_1}^2 = \frac{1}{N} \sum_j (Y_1^j)^2 = \frac{1}{N} \sum_{j=1}^N \langle \vec{J} \cdot \vec{l}_i \rangle^2 \text{ maximo da}$$

Lehen pausoa aurkitu nahi dugu:

\vec{l} \vec{Y}_1 -n bertsore direktzionala ?

(1) non: $S_{Y_1}^2 = \frac{1}{N} \sum_j \langle \vec{J} \cdot \vec{l}_i \rangle^2 = \vec{l}_i \cdot \mathbf{K} \cdot \vec{l}_i^T$ maximo da

$$S_{Y_1}^2 = \frac{1}{N} \sum_{j=1}^N \langle \vec{J} \cdot \vec{l}_i \rangle^2 = \frac{1}{N} \sum_{j=1}^N (L_{11}X_1^j + L_{12}X_2^j + \dots + L_{1p}X_p^j)^2 =$$

$$= \frac{1}{N} \sum_{j=1}^N \left(\sum_{k=1}^p \sum_{i=1}^d L_{ki} L_{1k} X_i^j \cdot X_k^j \right) = \sum_{k=1}^p \sum_{i=1}^d L_{1k} L_{1k} \frac{1}{N} \sum_{j=1}^N X_i^j \cdot X_k^j =$$

$$= \sum_{k=1}^p \sum_{i=1}^d L_{1k} L_{1k} \frac{1}{N} \langle \vec{X}_i \cdot \vec{X}_k \rangle = (L_{11} \ \dots \ L_{1p}) \begin{pmatrix} L_{11} \\ \vdots \\ L_{1p} \end{pmatrix} = \vec{l}_1 \cdot \mathbf{K} \cdot \vec{l}_1^T$$

* Metrika euklidearra gabiltzelako.

10/10/2014

10

Lagrange-ren funtzio definitzen badugu:

$$F = \frac{1}{N} \sum_{j=1}^N \langle \vec{J} \cdot \vec{L} \rangle^2 - \lambda (\|\vec{L}\|^2 - 1) = \frac{1}{N} \sum_j (X_1^j L_1 + \dots + X_p^j L_p)^2 - \left(\sum_{i=1}^p L_i^2 - 1 \right)$$

eta F deribatuz $L_1 L_2 \dots L_p$ ezezagunarekiko, ta zerora berdinuaz;
 (p + 1) ekuazioko sistema batekin aurkitzen gera:

$$S(1) : \begin{cases} \frac{\partial F}{\partial L_i} = \frac{1}{N} \sum_{r=1}^N (X_1^r + X_2^r L_2 + \dots + X_p^r L_p) X_i^r = 2 \lambda L_i = 0 \quad \forall i=1, \dots, p \\ \frac{\partial F}{\partial \lambda} = \sum_{i=1}^p L_i^2 - 1 = 0 \end{cases}$$

edo baliokide dena:

$$S'(1) : \begin{cases} \sum_{k=1}^p \frac{1}{N} L_k \sum_{j=1}^N X_k^j \cdot X_i^j = \sum_{k=1}^p L_k \text{ kob}(X_k X_i) = \lambda L_i \quad \forall i=1, \dots, p \\ \sum_{i=1}^p L_i^2 = 1 \end{cases}$$

eta matrirtzialki idatziaz:

$$S''(1) : \begin{cases} (L_1 \ L_2 \dots L_p) \begin{pmatrix} \dots k(X_1 X_i) \dots \\ k(X_2 X_i) \\ \vdots \\ k(X_p X_i) \end{pmatrix} = \lambda (L_1 \ L_2 \dots L_p) \quad \boxed{(L_1 \ L_2 \dots L_p)(K - I) = 0} \\ (L_1 \ L_2 \dots L_p) \begin{pmatrix} L_1 \\ L_2 \\ \vdots \\ L_p \end{pmatrix} = 1 \end{cases}$$

$$S''(1) : \begin{cases} (L_1 \ L_2 \dots L_p) \begin{pmatrix} k(X_1 X_i) \dots \\ k(X_2 X_i) \\ \vdots \\ k(X_p X_i) \end{pmatrix} = \lambda (L_1 \dots L_i \dots L_p) \\ S_y^2 = (L_1 \ L_2 \dots L_p) \begin{pmatrix} L_1 \\ L_2 \\ \vdots \\ L_p \end{pmatrix} = \lambda (L_1 \dots L_i \dots L_p) \begin{pmatrix} L_1 \\ L_2 \\ \vdots \\ L_p \end{pmatrix} = \lambda \end{cases}$$

erraz ikus daiteke orduan:

geure haserako (1) planteaimendua:

Zein dira : $(L_1 \dots L_p) ? , \lambda ?$

(1) non:
$$\begin{cases} (L_1 \dots L_p)(K - I) \lambda = 0 \\ \text{eta } S_y^2 = \lambda \text{ maximo da.} \end{cases}$$

Baina (1) sistema homogeneo honek hutsa ez den soluzio bat izango du $\forall \lambda / |K - \lambda I| = 0$; " λ "-ri K-ren autobalio propioa deritza.

Gure problema honetan datza: $K_{p \times p}^{x_1-x_p} = [K(x_i x_k)]$ kobariantza
 matritzaren edo aldagaiek tipifikatuak badira $R_{p \times p}^{z_1-z_p} = [r(z_i z_k)] = [k(z_i z_k)] = K_{p \times p}^{z_1-z_p}$
 (R = K) korrelazio matritzaren autobalio propioak aurkitu, (gaur egun ordinaroretaz erraz egitan da, ia (400 x 400) matritzaren autobalio propioak kalkulatzeko ordinaroretaz). K ala R, matritze simetrikoak direnez, dagozkien autobalio propioak desberdinak dira.

Orduan erresolbitu dugu (gehienetan ordinaroretaz):

$$\begin{vmatrix}
 k(x_1 x_p) - \lambda & x(x_1 x_2) & \dots & K(x_i x_p) \\
 K(x_i x_2) & K(x_2 x_2) - \lambda & \dots & k(x_2 x_p) \\
 \vdots & \vdots & \ddots & \vdots \\
 K(x_1 x_p) & K(x_1 x_p) & \dots & K(x_p x_p) - \lambda
 \end{vmatrix} = 0$$

$$|K - \lambda I| = 0$$

... eta suposa dezagun ekuazio berezi honen soluzioak ordenaturik hoiak direla:

$$\lambda_1 > \lambda_2 > \lambda_3 > \dots > \lambda_p > 0$$

non: $|K - \lambda_i I| = 0 \quad \forall \lambda_i$

Orduan hartuko dugu:

Lehenengo osagai nagusiaren bertsore direktzionaletaz: $\vec{L}_1 /$
 $\vec{L}_1 = (L_{11} \ L_{12} \dots \ L_{1p})$, kobariantza matritzaren λ_1 autobalio propio
 haundienari dagokion bektore propioa, hots: $(L_{11} \ L_{12} \dots \ L_{1p})(K - \lambda_1 I) = 0$
 betetzen duena.

a) $\vec{Y}_1 = L_{11}\vec{X}_1 + L_{12}\vec{X}_2 + \dots + L_{1p}\vec{X}_p$ lehenengo osagaiaren bektore
 adierazlea izango da.

b) "J" indibiduoak, Y_1 osagai nagusian hartzen duen balioa:

$$Y_1 = L_{11}X_1^j + L_{12}X_2^j + \dots + L_{1p}X_p^j \quad \forall J = 1 \text{ --- } N$$

c) Lehenengo osagai nagusiek jasotzen duen indibiduen sakaba-
 nakuntza edo bariantza:

$$S_{y_1}^2 = \frac{1}{N} \sum_j (Y_1^j)^2 = \lambda_1$$

Bigarren pauso batetan, hartuko dugu:

Bigarren osagai nagusiaren bertsore direktzionaletaz: $\vec{L}_2 /$
 $\vec{L}_2 = (L_{21} \ L_{22} \dots \ L_{2p})$, kobariantza matritzaren λ_2 bigarren autobalio
 propio haundienari dagokion bektore propioa, hots: $(L_{21} \ L_{22} \dots \ L_{2p})(K - \lambda_2 I) = 0$
 betetzen duena, zergatik $\vec{L}_2 \perp \vec{L}_1$ den?

Algebrak badakigu,

Matritze bat simetrikoa bada (geure kasuan K ala R simetrikoak
 dira), orduan matritzaren balio propio desberdinari dagozkien bektore
 propioak beren artean ortogonalak dira.

a) $\vec{Y}_2 = L_{21}\vec{X}_1 + L_{22}\vec{X}_2 + \dots + L_{2p}\vec{X}_p$ bigarren osagaiaren bektore
 adierazlea izango da. $\vec{Y}_2 \perp \vec{Y}_1$.

b) "J" indibiduoak Y_2 osagai nagusiaren hartzen duen balioa:

$$Y_2^j = L_{21} X_1^j + L_{22} X_2^j + \dots + L_{2p} X_p^j \quad \forall J = 1 \text{ --- } N$$

c) Bigarren osagai nagusiak jasotzen duen indibiduen sakaba-
 nakuntza edo bariantza:

$$S_{y_2}^2 = \frac{1}{N} \sum_j (Y_2^j)^2 = \lambda_2$$



r-garren osagai nagusiaren bertsore direkzioaletaz: $\vec{L}_r /$
 $\vec{L}_r = (L_{r1} L_{r2} \dots L_{rp})$, kobariantza matritzaren, λ_r , r-garren autobalio propio haundienari dagokion bektore propioa hartuko dugu.

Hots: $(L_{r1} L_{r2} \dots L_{rp})(K - \lambda_r I) = 0$ betetzen duena

Eta * -taz $\vec{L}_1 \perp \vec{L}_2 \perp \dots \perp \vec{L}_r$ izango da.

a) $\vec{Y}_r = L_{r1} \vec{X}_1 + L_{r2} \vec{X}_2 + \dots + L_{rp} \vec{X}_p$, r-garren osagaiaren bektore adierazlea izango da. $\vec{Y}_1 \perp \vec{Y}_2 \perp \dots \perp \vec{Y}_r$

b) "J" indibiduoak Y_r osagai nagusiaren hartzen duen balioa:

$$Y_r^j = L_{r1} X_1^j + L_{r2} X_2^j + \dots + L_{rp} X_p^j \quad \forall j = 1, \dots, N$$

c) r-garren osagai nagusiak jasotzen duen sakabanakuntza edo bariantza:

$$S_{Y_r}^2 = \frac{1}{N} \sum_j (Y_r^j)^2 = \lambda_r$$

IV. Osagai nagusien analisisan ezaugarri berezi bat zera da:

"Indibiduen sakabanakuntza osoa, p-osagai nagusiek jasotzen dutela"

Hots:

$$\sum_{j=1}^N \|J\|^2 = S_{X_1}^2 + S_{X_2}^2 + \dots + S_{X_p}^2 = \lambda_1 + \lambda_2 + \dots + \lambda_p$$

Orduan lehenengo r-osagai nagusiek, $Y_1 Y_2 \dots Y_r$, sakabanakuntzaren

$\frac{\lambda_1 + \lambda_2 + \dots + \lambda_r}{\sum_{i=1}^p \lambda_i}$ % jasotzen dute.

$$\sum_{i=1}^p \lambda_i$$

1998, p. 100.

1999, p. 100.

2000, p. 100.

2001, p. 100.

2002, p. 100.

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V. Zer esanahi emango diegu osagai nagusi bakoitzari, azterketan?

" \bar{Y}_i osagai nagusiaren esnahiak, ekarpen gehiena ematen dien aldagaien esanahiarekin zerikusi handia izango du".

*Gogora dezagun:

$$\bar{Y}_i = L_{i1}\bar{X}_1 + L_{i2}\bar{X}_2 + \dots + L_{ip}\bar{X}_p = r(Y_i X_1) \bar{X}_1 + r(Y_i X_2) \bar{X}_2 + \dots = r(Y_i X_p) \bar{X}_p \text{ dela.}$$

Adibidez:

$$\text{Baldin bada: } \bar{Y}_i = 0,19 \bar{X}_1 + \underline{0,98} \bar{X}_2 + 0,36 \bar{X}_3 + \dots - \underline{0,85} \bar{X}_p$$

Orduan:

\bar{Y}_i -n ekarpen gehiena duten aldagaiak X_2 , X_p dira, batek ekarpen positiboa eta besteek negatiboa.

4. "Pentsa dezagun Euskal Herriko egoerataz ikerketa bat egin nahi dugula":

- Aurkezpen bezala 140 herri aukeratu ditugu.

Herri hoiak errepresentagarri izan daitezten, metodo arrazional bat jarraitu behar dugu aukeratzekoan (aurkezpen teoria oso lagungarri zaigu urrats honetan).

- Aurkezpeneko herri bakoitzean 40 test zenbatugarri egin ditugu, eta suposa dezagun balioak ondoko taulan azaltzen direnak direla:

	1	2	3	...	J	...	140
x_1	20	500	30	...	x_1^j	...	80
x_2	5	30	5	...	x_2^j	...	4
x_{40}	90	50	70	...	x_{40}^j	...	25

x_1 = telefonoen kopurua / mila biztanlekiko

x_2 = bi liburu baino gehiago urtean zeinek irakurtzen dituen / ehun biztanlekiko.

x_3 = renta per kapita

⋮

x_{40} = Ehunetik zenbat diren etxearen jabe.

Lehen urratsean: taula tipifikatu egingo dugu, testak unitate desberdinetan neurtuak baitira.

$$\begin{array}{c}
 \begin{array}{cccc}
 & 1 & 2 & 3 & & J & & 140 \\
 \hline
 Z_1 & & & & & Z_1^J & & \\
 Z_2 & & & & & Z_2^J & & \\
 \vdots & & & & & \vdots & & \\
 Z_{40} & & & & & Z_{40}^J & &
 \end{array}
 \end{array}$$

$(Z_i^J > 0 \quad \text{ala} \quad Z_i^J < 0 \quad \text{ala} \quad Z_i^J = 0)$

Bigarren urratsean: korrelazio matritza kalkulatu dugu: Suposa dezagun honela dela:

$$R_{40 \times 40}^{Z_1-Z_{40}} = \begin{bmatrix}
 1 & 0,8 & -0,7 & \dots & 0,3 \\
 0,8 & 1 & 0,5 & \dots & 0,8 \\
 -0,7 & 0,5 & 1 & \dots & 0 \\
 \vdots & \vdots & \vdots & \ddots & \vdots \\
 0,3 & 0,8 & 0 & \dots & 1
 \end{bmatrix} = K_{40 \times 40}^{Z_1-Z_{40}}$$

(Obserba genezake korrelazio handia dagoela aldagaien artean).

Hirugarren urrtsean; ordinarretaz, korrelazio matritzaren auto-balio propioak kalkula genezake.

Suposa dezagun hoiek direla:

$$- \lambda_1 = 3,45 > \lambda_2 = 2,15 > \lambda_3 = 0,40 > \dots > \lambda_{40} = 0,001$$

... eta $\sum_{i=1}^{40} \lambda_i = 3,45 + 2,15 + \dots + 0,001 = 7,1$ baldin bada:

- lehenengo bi osagai nagusiek sakabanakuntzaren %80 jasotzen dute.

(Hots: $\frac{3,45 + 2,15}{7,1} = 0,8$)

* Hau ez da oso erreala, praktikan sakabanakuntzaren %80 jasotzeko bi osagai nagusi bainan gehiago kontuan hartu behar dira, bainan exemplu honetan honela suposatuko dugu. (errestako).

Laugarren urratsean: ordinadoretaz, korrelazio matritzaren bi bektore propio kalkulatu ditugu, $\lambda_1 = 3,45$, $\lambda_2 = 2,15$ balio propioerik dagozkieenak.

Orduan lehen bi osagai nagusien bektore adierazleak ezagunak izango zaizkigu.

Hots:

1. $(L_1 \ L_2 \dots L_{40})(R - 3,45 I) = 0$ sistema homogeneoa askatuaz:
 suposa dezagun: $1 = (l_{11} \ l_{12} \dots \ l_{140}) = (0,60, 0,15, 0,98, \dots, 0,69)$ dela
 orduan: $\vec{Y}_1 = 0,60 \vec{Z}_1 + 0,15 \vec{Z}_2 + 0,98 \vec{Z}_3 + \dots + 0,69 \vec{Z}_{40}$

2. $(L_1 \ L_2 \dots L_{40})(R - 2,15 I) = 0$ sistema homogeneoa askatuaz:
 suposa dezagun: $2 = (l_{21} \ l_{22} \dots \ l_{240}) = (0,25, 0,70, 0,10, -0,80, \dots, 0,10)$
 orduan: $\vec{Y}_2 = 0,25 \vec{Z}_1 + 0,70 \vec{Z}_2 + 0,10 \vec{Z}_3 - 0,80 \vec{Z}_4 + \dots = 0,10 \vec{Z}_{40}$

Bostgarren urratsean: esanahi bat ematen saiatuko gara osagai nagusierik. Urrats honetan, noski espezialisten laguntza behar beharrezkoa zaigu (soziologo, ekonomilari, sendagile...)

Exenplu honetan: $\vec{Y}_1 = 0,60 \vec{Z}_1 + 0,15 \vec{Z}_2 + 0,98 \vec{Z}_3 + \dots + 0,69 \vec{Z}_{40}$ bada.

Y_1 -n zein aldagaiek dute ekarpen gehiena?

ordenaturik $Z_3, Z_{40}, Z_1 \dots$

Orduan: Y_1 osagai nagusiak esan genezake herrien maila ekonomikoarekin zerikusi handia duela.

Exenplu honetan: $\vec{Y}_2 = 0,25 \vec{Z}_1 + 0,70 \vec{Z}_2 + 0,10 \vec{Z}_3 - 0,80 \vec{Z}_4 + \dots + 0,10 \vec{Z}_{40}$ bada

Y_2 -n zein aldagaiek dute ekarpen gehiena?

ordenaturik: Z_4, Z_2, \dots

Orduan: Y_2 osagai nagusiak esan genezake herrien maila kulturalarekin zerikusi handia duela.



Seigarren urratsean: gure aurkezpeneko 140 herriak, Y_1, Y_2 osagaietan hartzen dituzten balioak kalkula genezake.

Hots:

$$Y_1^j = 0,60 Z_1^j + 0,15 Z_2^j + \dots + 0,69 Z_{40}^j$$

$$Y_2^j = 0,25 Z_1^j + 0,70 Z_2^j + \dots + 0,10 Z_{40}^j$$

$$\forall j : (Y_1^j, Y_2^j)$$

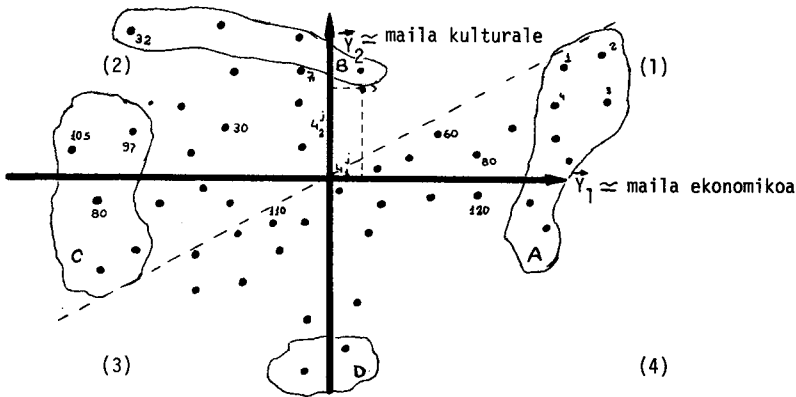
Zazpigarren urratsean; taula batez eta geometrikoki azalduko ditugu;

(A)

Y_1	Y_1^1	Y_1^2 —————	Y_1^j —————	Y_1^{100}	$\bar{Y}_1 = 0; S_{y_1}^2 = \lambda_1 = 3,45$
Y_2	Y_2^1	Y_2^2 —————	Y_2^j —————	Y_2^{100}	$\bar{Y}_2 = 0; S_{y_2}^2 = \lambda_2 = 2,15$

1, 2, ————— J, ————— 100

(B)



Azterketaren ondorio nabarienenak:

- a) 1. arloan kokatu diren herriak maila ekonomiko eta maila kulturala mediatikan gora dute.
2. arloan kokatu diren herriak, maila ekonomikoa mediatikan behera dute, eta maila kulturala mediatikan gora.
3. arloan kokatu diren herriak maila ekonomikoa eta maila kulturala mediatikan behera dute.



4. arloan kokatu diren herriak maila ekonomikoa mediatikan gora dute eta maila kulturala mediatikan behera.

- b) Maila ekonomikoa handiena duten herriak A multzokoak dira.
 - c) Maila kulturala handiena duten herriak B multzokoak dira.
 - d) Maila ekonomiko txikiena duten herriak C multzokoak dira.
 - e) Maila kultural txikiena duten herriak D multzokoak dira.
 - f) Maila ekonomikoa kulturala baino handiagoa da Euskal Herrian zeren eta ia herri guztiak $Y_1 > Y_2$ arloan kokatzen dira.
 - g) Nola explika "82" herriaren egoera?
 - h) Hurbil kokatzen diren herriak, egoera berdintsukoak dira ...
- ... Eta honela jarrai genezake. ...